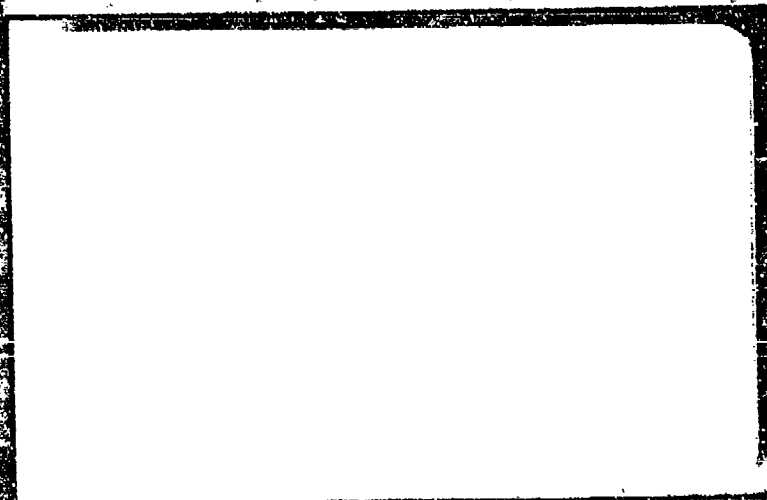




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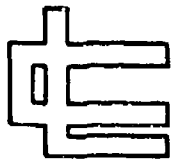
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A REVISITATION OF THE PHENOMENOLOGICAL  
APPROACH WITH APPLICATIONS TO  
RADAR TARGET DECOMPOSITION

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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>Basic concepts of radar target decomposition in terms of the Mueller matrix<br>formulation are considered. This special report provides a more detailed<br>interpretation of the author's original theory developed in his dissertation.<br>For details: see reverse side. |                                      |  |

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## 20. ABSTRACT

This report highlights some results of a phenomenological approach to radar targets, with applications. The approach grew out of the common sense realization that only those target data are acceptable for discrimination and identification purposes which can be shown to relate in a physically meaningful way to basic target structure. Only then can data, often gathered at great expense, obtained for one type of system, be expected to be useful productively for a new system and hence improve efficiency and cost factors.

Although these comments are almost self-evident and common sensical in nature, examples are given to show how this systematic approach has an important effect on the mathematical and practical development toward target identification (inverse) problems. The effect of antenna and target orientation angle (Fig. 1) on corrupting target information is stressed, in contrast to common practice to allow single H or V polarization data to be accepted as meaningful. So-called orientation-independent target parameters are derived from the target Stokes matrix which allows for physical interpretation and correlation with target structure.

The report summarizes the general target decomposition theorems, proved by the author in 1970. It shows that a single-coherent object is electromagnetically irreducible (it cannot be broken down mathematically as the incoherent sum of the smaller parts without violating physical principles). A general distributed target (such as chaff or surface return) can be broken down into the (incoherent) sums of a "single average object" and "N-target" residue. The latter may be considered as a form of "target noise".

All this opens up new vistas for optimal signal processing schemes which extend the present predominantly scalar case to include vector scattering problems. It is hoped that by these efforts improved reliability with reduced costs for target discrimination and identification purposes can be achieved.

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A REVISITATION OF THE PHENOMENOLOGICAL APPROACH  
WITH APPLICATION TO RADAR TARGET DECOMPOSITION \*

by

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1. Introduction

The historical development of radar target polarimetry can be roughly divided into two different types of efforts. The first type consists of providing the general tools for analysis and data acquisition, which are aimed at satisfying immediate and specific systems requirements for radar cross section (RCS) data. These methods were developed mostly before and during the 1960's and are still continuing today.

The main thrust of these efforts was provided by the realization that only "complete polarization data" are adequate to present RCS. The so-called null-polarization plots are the culmination of this approach, because null-plots are equivalent to complete scattering matrix information, which can be used for dynamic target analysis.

A second approach grew out of the gradual realization that the availability of "complete polarization data" by itself does not guarantee a better understanding of measured parameters in relationship to the radar targets concerned. What was needed instead was a fundamental new approach or attitude towards studying data relevance, with reference to the targets concerned.

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This approach, which was called "phenomenological" by Huynen, was developed systematically in his Ph.D. thesis (Huynen, 1970) and more recently in [5] (Huynen, 1978). This approach required, among other things, a slightly different mathematical presentation, which followed closely the natural geometrical concepts, which underly the interaction between target scattering and antennas for transmitting and receiving. Because the full significance of the phenomenological method is not yet sufficiently understood, it was felt useful to give the following expose below. We will illustrate the method by giving several examples. For the purpose of contrast, we will call the conventional approach "data processing oriented", and we will show how a phenomenological approach differs from a purely data processing-oriented one. Most of the ideas presented are almost self-evident once they are understood, yet it is surprising to find in practical experience how many sins against common sense are often committed. It is from the background of that reality that it was felt useful and necessary to provide a systematic method for analyzing radar targets.

## 2. The Received Voltage

The equation which gives received voltage  $V(t)$  from a radar target, with scattering matrix  $[S(t)]$ , obtained by transmitting an elliptically polarized wave  $\underline{h}_T$  and using a receiver antenna with receiver-polarization  $\underline{h}_R$  is given by

$$V(t) = [S(t)] \underline{h}_T \cdot \underline{h}_R \quad (1)$$

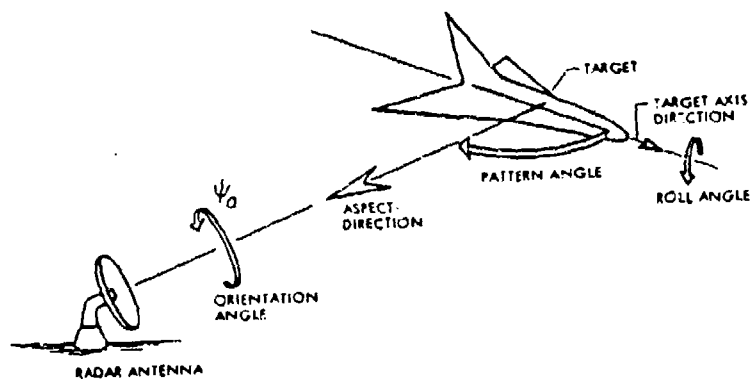


Fig. 1 . Target aspect direction and orientation angle.

All possible information that can be derived from the target is given by this deceptively innocent looking equation (1). The equation looks simple enough, yet it represents the very complex interacting processes between target structure and electromagnetic wave structure. Thus all factors regarding frequencies, polarizations, wave forms, antenna gain patterns, target structure, composition and shape, target position in space and time, aspect angles, orientation and roll-angles (see Fig. 1), atmospheric effects, environmental interactions, noise, turbulence, etc. are incorporated into the equation. In other words, one can interpret the simple equation as representing a vast and complex radar target technology, with associated signal processing, applicable to each special case.

In fact, one can look at the equation from a strictly data processing point of view and consider any data based on equation (1) as potentially useful information regarding the target. It then follows from the same logic that more data will produce more information, and hence potentially more useful parameters for target discrimination and identification will become available. This has been a traditional approach and the argument that the full knowledge of the scattering matrix  $[S]$  represents complete electromagnetic information is considered most rightly, by the data processing people, as a most desirable objective to be aimed for and supported. Essentially their philosophy may be expressed succinctly as follows: The more data one can accumulate from a target, the more accurate the final identification is bound to be!

As we shall see, however, the problem of target discrimination cannot be

solved by such simplistic approaches. Due to the variety and vast number of parameters that can characterize a set of given targets, one soon overloads the data processing capacity of the system and the procedure becomes excessively cumbersome, time-consuming, and costly.

What is even worse is that for every new case under investigation with new targets, new environments, new frequencies and new systems demands, the whole procedure has to be repeated all over again because there is no obvious way that the information, often gathered at great expense, can be applied to the new case. This is because one has no clear understanding how the measured data correlate with target structure.

It was for these reasons that a new approach to the whole problem of radar target discrimination and identification was considered necessary. This approach was called "phenomenological" because it aims at keeping the primary phenomenon — in this case the properties of radar target structure — in continual focus of the observers' attention. We will now illustrate how this seemingly obvious approach is contrasted to the traditional data processing one.

### 3. Two Different Approaches to Radar Target Analysis

Let us again consider equation (1). The data processor looks at the left hand side of the equation, the received voltage, as the primary source of received information from the target. But we see at once that this information consists of a mixed bag of target properties and antenna properties.

And ultimately we are interested in the former, not the latter. From the phenomenological point of view, it is of the utmost importance to try to unravel the entanglement of antenna and target properties before serious attempts at target discrimination and identification are undertaken.

Equation (1) clearly distinguishes between antenna polarization  $\underline{h}_T$  and  $\underline{h}_R$  and target observables, represented by scattering matrix  $[S(t)]$ . Hence, one often hears a target being described by its "polarization properties".

Almost all practical targets produce a different voltage when illuminated by horizontal polarization (H) or with vertical (V). This is due to the electromagnetic interaction of the two-dimensional wave structure and target structure. In the phenomenological approach, we have to consider the question: How does this information relate to the target as a physical object?

Suppose someone cocks his head ninety degrees sideways, always looking straight ahead at a fixed object, and claims he now observes a different object! We would immediately question his interpretation because we have learned, through experience in early infancy (see J. Piaget: The early development of the child) that real objects in space have an invariant property regarding changes in orientation angle (cocking one's head sideways), keeping target exposure (line of view towards the target, see Fig. 1) otherwise fixed. Our brain manages to compensate for this change in image on the retina, and we thus observe the same object in space. If we did not possess this ability, a bewildering variety of objects would be presented to us, anytime

we slightly moved our orientation angle towards the object, and discrimination and identification of ordinary objects in space would become an impossible task.

Now, the electromagnetic interaction with radar targets violates this requirement of orientation-invariance because target illuminations with H and V polarizations, at the same exposure angle, do give different signatures. Faced with these facts, we have to conclude that these two pieces of data violate the property that meaningful target information must be orientation-invariant, and hence the data obtained cannot be admitted immediately as a target discriminant. Thus, while the data processor would admit the data as meaningful, the phenomenologist would reject the data as being "incomplete". We thus find that the drive towards more complete data has an altogether different origin in the two approaches!

The data processor welcomes any new piece of information he can use in his algorithms to achieve his objectives, whereas the phenomenologist tries to filter out only those significant data which relate directly to distinctive target behavior. By doing so he hopes to greatly simplify the amount of data processing required and to increase discrimination accuracy by concentrating on a few meaningful parameters which are correlated to target structure, and which information can be used and generalized to new cases.

These almost obvious remarks have, nevertheless, serious consequences on the mathematical and practical development towards radar target discrimination and identification techniques, as we shall see shortly.

#### 4. The Description of Antenna Polarization

To give an illustrative example: Antenna polarization is almost universally written in the literature as a two-component complex vector as follows

[1], [6]:

$$\underline{h} = h_A \hat{h}_A + h_B \hat{h}_B = \begin{bmatrix} h_A \\ h_B \end{bmatrix}_{AB} \quad (2)$$

where  $\hat{h}_A$  and  $\hat{h}_B$  represent a general orthonormal basis. Most commonly used are the linear (H, V) or a circular basis (right- and left-circular unit vectors). The general basis (2) has an advantage that one can present equations in a general form and one can use an appropriate basis, to be chosen later on. This is done traditionally to derive useful and elegant expressions which enable one to calculate, for example, the target's co-nulls and x-null polarizations. However, the generality of equation (2) also presents disadvantages as we shall see shortly.

What equation (2) describes is a two-dimensional elliptically polarized wave, which propagates through space to or from the target. Hence, its geometric significance is the ellipse shown in Fig. 2.

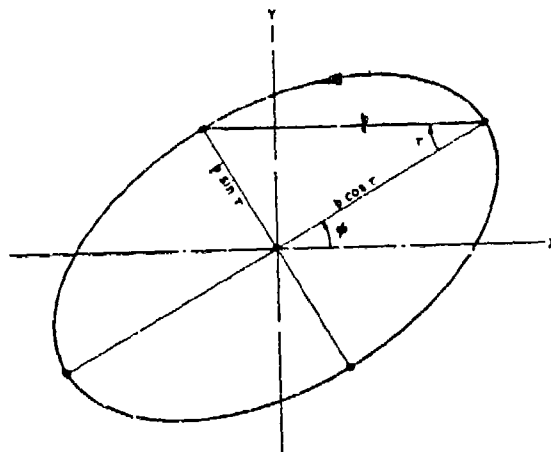


Fig. 2. Left-sensed polarization ellipse in fixed plane.

The full geometric information is contained by the total amplitude  $p$  of the wave, the ellipticity angle  $\tau$ , and the orientation angle  $\phi$  of the ellipse. From the discussion above, it should be evident that the orientation  $\phi$  of the wave, as it moves towards the target, is of direct importance for target discrimination purposes because it represents a measure of significance related to target structure. If target returns are dependent on orientation angle  $\phi$  — keeping target exposure otherwise fixed — and most targets are, then these data cannot signify a meaningful target discriminant which relates to target structure. This is because, as we found above, meaningful target structure parameters have to be orientation-independent if target exposure otherwise is kept the same. This is not to say that in a dynamic situation orientation may be important as a dynamic (but not structural) target parameter.

Hence we cannot use equation (2) to formulate our search for target structure discriminants, because equation (2) does not display the antenna polarization orientation parameter  $\phi$ . Obviously, it is quite legitimate to start with equation (2) and at a later state of the calculation convert mathematically to the desired geometrical parameters, but experience has shown that this is rarely done in practice on a consistent basis.

Much confusion could have been avoided if one simply wrote the transmitted wave in terms of geometric parameters, as follows:

$$\underline{h}(a, \alpha, \phi, \tau) = ae^{i\alpha} \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \tau \\ i \sin \tau \end{bmatrix} \quad (3)$$

where  $\alpha$  is an absolute phase factor.

We quote here from C.-Y. Chan's master thesis [2]:

One advantage of using the geometric parameter representation is that by specifying the set of parameters for a particular polarization, it gives us a pictorial impression about the rough sketch of the polarization ellipse. For example, if  $\tau=22.5^\circ$  and  $\phi=45^\circ$ , we can immediately visualize the picture of an ellipse whose ellipticity is halfway between linear ( $\tau=0$ ) and circular ( $\tau=\pm 45^\circ$ ), the wave is left-handed and the major axis of the ellipse makes an angle of  $45^\circ$  with respect to the horizontal reference axis. We cannot get such a mental picture if other representations were used.

Table 1      Examples of polarizations expressed in  
geometric parameters

| <u>Polarization</u>     | <u><math>E(a, \phi, \tau)</math></u> |
|-------------------------|--------------------------------------|
| Linear                  | $\tau=0$                             |
| Horizontal              | $\tau=0, \phi=0$                     |
| Vertical                | $\tau=0, \phi=\pm 90^\circ$          |
| Left-handed circular    | $\tau=45^\circ$                      |
| Right-handed circular   | $\tau=-45^\circ$                     |
| Left-handed elliptical  | $0 < \tau < +45^\circ$               |
| Right-handed elliptical | $-45^\circ < \tau < 0$               |

##### 5. The Significance of the Relative Target Orientation Parameter

We have stressed the significance of antenna orientation  $\phi$  because it relates directly to the target orientation parameter  $\psi$ . We will show shortly that every target at a given exposure and frequency has its own natural orientation  $\psi$ . For a roll-symmetric object, this angle is simply related to the target roll axis, but any odd-shaped target has its unique orientation  $\psi$ . As the target moves, each exposure produces a different orientation  $\psi$ , which may depend on frequency (see Fig. 1).

The target orientation angle  $\psi$  is easily calculated from the scattering matrix  $[S]$ ; it is the orientation of the ellipse associated with the maximum

polarization or x-nulls, which is characteristic for the target. It should be clear by now that the only relevant orientation, as far as the target's illumination is concerned, is the relative orientation  $\Phi = \phi - \psi$ .

Again we observe the importance of keeping track of  $\phi$  and  $\psi$ , and their difference, by using the proper mathematical framework for antennas and target. If this is not done, a bewildering variety of mathematical forms may arise, where  $\phi$  and  $\psi$  appear separately, not as a difference, with, as a result, total confusion in abundance. The phenomenological, and plain common sense, approach requires that only the relative orientation plays a role in the formalism.

#### 6. The Representation of the Target Scattering Matrix

We now will show how these almost self-evident concepts are incorporated into the mathematical scheme. If we start with the most general polarization basis (AB), which led to (2), then the target matrix is simply the collection of four complex numbers [1]

$$[S(AB)] = \begin{bmatrix} S_{AA} & S_{AB} \\ S_{BA} & S_{BB} \end{bmatrix} \quad (4)$$

A unitary transformation  $[U(AB; A'B')]$  applied to (4) results in the matrix  $[S]$  presented in a new basic frame  $(A'B')$ :

$$[S(A'B')] = [U^T][S(AB)][U] \quad (5)$$

Matrix  $[U]$  can be given a particularly simple form:

$$[U] = \frac{1}{\sqrt{1 + |\rho|^2}} \begin{bmatrix} 1 & -\rho^* \\ \rho & 1 \end{bmatrix} \quad (6)$$

where  $\rho$  is the complex polarization ratio.

The coefficients of  $S(A'B')$ , based on (5) are found as [1]:

$$S_{A'A'} = (1 + |\rho|^2)^{-1} (S_{AA} + \rho^2 S_{BB} + 2\rho S_{AB}) \quad (7)$$

$$S_{A'B'} = (1 + |\rho|^2)^{-1} (-\rho^* S_{AA} + \rho S_{BB} + (1 - |\rho|^2) S_{AB}) \quad (8)$$

$$S_{B'B'} = (1 + |\rho|^2)^{-1} (\rho^* S_{AA} + S_{BB} - 2\rho^* S_{AB}) \quad (9)$$

We may use for  $[U]$  the orthonormal basis vectors  $[U] = [\underline{m}, \underline{m}_\perp]$ , where  $\underline{m} = \underline{h}_m$ ,

the so-called maximum polarization or x-polarization null, is the eigenvector solution of the characteristic eigenvalue problem for  $[S(AB)]$ :

$$[S(AB)]\underline{x} = \lambda \underline{x}^* \quad (10)$$

Because of the orthonormal properties of  $\underline{m}$  and  $\underline{m}_\perp$ , which satisfy (10), we find the condition that the off-diagonal term  $[S_{A'B'}]$  in (8) becomes zero, and this fact can be used in turn to solve for  $\rho$  and hence for  $\underline{m}$  and  $\underline{m}_\perp$ , without solving the eigenvalue problem (10) directly.

We now write for  $\underline{m} = h_m$  in geometrical variables:

$$\underline{m}(\psi, \tau_m) = \begin{bmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{bmatrix} \begin{bmatrix} \cos \tau_m \\ i \sin \tau_m \end{bmatrix} \quad (11)$$

For the complex valued eigenvalues  $\delta_1$  and  $\delta_2$  which satisfy (10) we write:

$$\begin{aligned} \delta_1 &= m e^{2i(\nu + \beta)} \\ \delta_2 &= m \tan^2 \gamma e^{-2i(\nu - \beta)} \end{aligned} \quad (12)$$

We have now a complete description of the scattering matrix  $[S(AB)]$  in terms of geometrical target parameters. From (5) we obtain:

$$[S(AB)] = [U^*(\underline{m}, \underline{m}_\perp)] \begin{bmatrix} m e^{2i(\nu + \beta)} & 0 \\ 0 & m \tan^2 \gamma e^{-2i(\nu - \beta)} \end{bmatrix} [U^*(\underline{m}, \underline{m}_\perp)]^T \quad (13)$$

The geometrical parameters are:

$m$ ,  $\gamma$ ,  $\psi$ ,  $\tau_m$ ,  $\nu$  and  $\beta$ . The positive quantity  $m$  denotes target magnitude - it may be viewed as an overall measure for target size. The angle  $\gamma$  is the characteristic angle, it determines separation of the targets co-nulls on the polarization sphere. The angle  $\psi$  is the celebrated target orientation angle which we showed is a dynamic variable.

As soon as the angle  $\psi$  is found, it can be separated from all other target parameters and hence these target parameters are orientation independent (but are still dependent on target exposure and frequency, etc.) and can be

used to characterize target structure! Without the basic mathematical framework (13) in which to express  $[S(AB)]$ , it would not have been possible to compute  $\psi$  and at once to eliminate it from the other target parameters.

Also, it is easily seen that the combination of equation (3), (11), and (13) into (1) results in only the relative orientation  $\Phi = \phi - \psi$  having significance in the target return, as required by common sense.

The three angles  $\psi$ ,  $\tau_m$  and  $\nu$  are simply the Eulerian rotation angles about three orthogonal axes (see Fig. 3, which is the geometrical equivalent to equation (13)). Finally  $\beta$  is the absolute phase of the target, it disappears with power measurements.

Aside from its geometrical significance, the Eulerian angles are also powerful indicators of target structure:  $\nu$  is called the skip-angle because it relates to double bounce scattering,  $\tau_m$  is the helicity-angle and is a powerful indicator of target symmetry or non-symmetry (for symmetric targets,  $\tau_m = 0$ ).

All this information, and much more to come, would have been impossible to extract if we blindly followed some general sort of data processing scheme without keeping a mental focus on data relevance to target structure.



The nomenclature given by (15) supplies exactly the same information as in (3), with  $g_o = a^2$ , except that the absolute phase  $\alpha$  disappears with power measurements. Now the Stokes matrix  $[M]$  in (14) can be made equally analogous to the scattering matrix  $S(AB)$  in (13). We thus find:

$$[M] = \begin{bmatrix} A_o + B_o & C & H=0 & F \\ C & A_o + B & E & G \\ H=0 & E & A_o - B & D \\ F & G & D & -A_o + B_o \end{bmatrix} \quad (16)$$

where

$$\begin{aligned} A_o &= Q f \cos^2 2\tau_m \\ A_c &= Q(1 + \cos^2 2\gamma - f \cos^2 2\tau_m) \\ B &= Q(1 + \cos^2 2\gamma - f(1 + \sin^2 2\tau_m)) \\ C &= 2Q \cos 2\gamma \cos 2\tau_m \\ D &= Q \sin^2 2\gamma \sin 4\psi \cos 2\tau_m \\ E &= -Q \sin^2 2\gamma \sin 4\psi \sin 2\tau_m \\ F &= 2Q \cos 2\gamma \sin 2\tau_m \\ G &= Q f \sin 4\tau_m \end{aligned} \quad (17)$$

$$Q = \frac{P_A^2 P_B^2 m^2}{8 \cos^4 \gamma} \quad (18)$$

$$f = 1 - \sin^2 2\gamma \sin^2 2\psi \quad (19)$$

Notice that the target orientation angle  $\psi$  does not appear in the defining equations (17, 18, 19). Instead it has been incorporated with the antennas  $g_T$  and  $g_R$ , where it appears as  $\Phi = \phi - \psi$  instead of  $\phi$  in (15). This was an essential requirement for our approach to orientation-independent target

discriminants. Hence all target parameters in (17) are orientation independent. It would be an easy matter to transform  $\psi$  back into the Stokes matrix (16). We would then obtain

$$[M_{\psi}(t)] = \begin{bmatrix} A_0 + B_0 & C_{\psi} & H_{\psi} & F \\ C_{\psi} & A_0 + B_0 & E_{\psi} & G_{\psi} \\ H_{\psi} & E_{\psi} & A_0 - B_0 & D_{\psi} \\ F & G_{\psi} & D_{\psi} & -A_0 + B_0 \end{bmatrix} \quad (20)$$

where:

$$H_{\psi} = C \sin 2\psi \quad (21)$$

$$C_{\psi} = C \cos 2\psi$$

$$G_{\psi} = G \cos 2\psi - D \sin 2\psi \quad (22)$$

$$D_{\psi} = G \sin 2\psi + D \cos 2\psi$$

$$E_{\psi} = E \cos 4\psi + B \sin 4\psi \quad (23)$$

$$B_{\psi} = -E \sin 4\psi + B \cos 4\psi$$

We notice in particular that for a target on axis ( $\psi=0$ ),  $H_{\psi}=0$ , as shown in (16). This result would not be obvious from generalistic considerations. The author has yielded to the political argument that (15) is the form for the Stokes vector most often used in optics [1]. However, in radar polarimetry the system [I, V, Q, U] seems more natural, as can be seen from (16):  $H=0$  appears in a non-symmetric and illogical place inside the matrix. Using the authors original notation [3, 5] would place H in the upper right-hand corner in a better symmetrical position. A similar argument can be made for a better symmetrical placement of G, which indicates coupling between the targets' symmetric and nonsymmetric components (see later). Nevertheless,

in this report the formalism (15) is used. It is usually an easy matter to transpose indices and convert to any system desirable.

We notice that  $[M]$  and  $[M_\psi]$  are symmetric matrices (not to be confused with symmetric targets!), and that they obey an important trace rule

$$\text{trace } \{[M_\psi]\} = 2(A_0 + B_0) \quad (24)$$

We also know that the conventional scattering matrix (4) is given by five independent parameters (we excluded absolute phase). Now the Stokes matrix (20) for the single object has exactly the same content as (4), but in contrast it shows nine parameters which thus cannot be all independent. Hence there must be four dependency relationships. On the other hand for distributed targets (see later discussion) due to the averaging process on each matrix coefficient, the distributed target is given in general by nine independent parameters. These facts will play a fundamental role with the target decomposition theorems applicable to distributed targets.

#### 8. Discussion of Target Parameters

First we consider reception for circular polarization combinations, based upon (14). There are three cases designated by (RC, RC), (LC, LC) and (RC, LC). We find for received power

$$P(LC, LC) = 2(B_0 + F) \quad (25)$$

$$P(RC, RC) = 2(B_0 - F) \quad (26)$$

$$P(RC, LC) = 2A_0 \quad (27)$$

From these equations, it follows that  $A_0 \geq 0$  and  $B_0 \geq 0$ . These two parameters are basic to target structure.  $A_0$  is associated with regular, smooth, convex type of surface scattering, which contributes to specular returns (for a sphere  $A_0$  is the only non-zero parameter). On the other hand,  $B_0$  may be considered as a measure of all the target's non-symmetric, irregular, rough-edged, non-convex depolarizing components of scattering. The orientation-independent parameter  $F$  is characteristic for right or left wound helices viewed on axis. Eqn. (17) shows that  $F$  is proportional to  $\sin 2\tau_m$ . For a roll-symmetric object, there can be no preference for LC or RC polarizations, hence from (25) and (26) we find  $F=0$  and  $\tau_m=0$  are sensitive indicators of target symmetry.

Parameter  $C$  is related to eccentricity of target shape:  $C=0$  for a sphere and  $C \neq 0$  for a wire target. The role of  $D$  remains obscure.

Most of these results have been reported elsewhere [3] and [5], but because of lack of familiarity, it was thought important and useful to summarize some of the highlights. We will continue the discussion of target parameters in Section 10.

#### 9. Conditions for single (coherent) radar targets

A very important and useful condition imposed on the Stokes matrix  $[M]$  for

single targets will now be derived. The term "single" is used here in opposition to a multiple set of independent targets or a distributed target. This result, which is basic to the target decomposition theorems to be discussed later, was derived in 1970 by Huynen [3], but no follow-up work is known to have appeared on this in the literature. The basic idea here is that of a single (later we shall see: irreducible) target, which produces at each instant coherent scattering.

The desired result is a consequence of the definition of a single object with scattering matrix  $[M]$ . Let  $\delta = [M]g$  be the scattered wave, then  $\delta$  will be coherent only if  $\delta_0^2 = \delta_1^2 + \delta_2^2 + \delta_3^2$  is satisfied. In matrix form we have

$$\delta_0^2 - \delta_1^2 - \delta_2^2 - \delta_3^2 = [Z]\delta \cdot \delta = 0 \quad (28)$$

where

$$[Z] = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}. \quad \text{If we substitute } \delta = [M]g \text{ into (28) we obtain}$$

$$([M][Z][M]g) \cdot g = [Q]g \cdot g = [Q_M]g \cdot g = 0 \quad (29)$$

The matrix  $[Q] = [M][Z][M]$  will have the following form

$$[Q] = \begin{bmatrix} Q_{00} & Q_{01} & Q_{02} & Q_{03} \\ Q_{10} & Q_{11} & Q_{12} & Q_{13} \\ Q_{20} & Q_{21} & Q_{22} & Q_{23} \\ Q_{30} & Q_{31} & Q_{32} & Q_{33} \end{bmatrix} \quad (30)$$

Because of symmetry,  $Q_{ij} = Q_{ji}$  if  $i \neq j$ . Substituting [M] from (16) or (20) gives the following terms (with index  $\psi$  removed):

$$\begin{aligned}
 Q_{01} &= (B_o - B)C - (EH + FG) \\
 Q_{23} &= (B_o - B)D - (EG - FH) \\
 Q_{02} &= -(B_o + B)H - (DF + CE) \\
 Q_{12} &= -(B_o + B)G + (CF - DE) \\
 Q_{03} &= 2A_o F - (CG + DH) \\
 Q_{13} &= -2A_o E + (CH - DG)
 \end{aligned} \tag{31}$$

For the diagonal terms we write:

$$\begin{aligned}
 Q_{00} &= \frac{1}{2}(-Q_0 + Q_1 + Q_2 + Q_3) \\
 Q_{11} &= \frac{1}{2}(Q_0 - Q_1 + Q_2 + Q_3) \\
 Q_{22} &= \frac{1}{2}(Q_0 + Q_1 - Q_2 + Q_3) \\
 Q_{33} &= \frac{1}{2}(Q_0 + Q_1 + Q_2 - Q_3)
 \end{aligned} \tag{32}$$

where

$$Q_1 = 2A_o(B_o + B) - (C^2 + D^2) \tag{33}$$

$$Q_2 = 2A_o(B_o - B) - (G^2 + H^2) \tag{34}$$

$$Q_3 = (B_o^2 - B^2) - (E^2 + F^2) \tag{35}$$

and

$$Q_o = -2A_o^2 - B_o^2 - B^2 + C^2 + D^2 + E^2 + F^2 + H^2 + G^2 \tag{36}$$

First we observe that [Q] contains a term  $-\frac{1}{2}Q_o[Z]$ , which does not in any way contribute to (29) because  $[Z]g \cdot g = 0$  (the transmit antenna is coherent). This means that the term with  $Q_o$  should be dropped from [Q], which produces the desired matrix  $[Q_M]$ :

$$[Q_M] = [Q] + \frac{1}{2}Q_0[Z] \quad (37)$$

Now  $[Q_M]$  satisfies the trace rule, which was characteristic for  $[M]$

$$\text{trace}\{[Q_M]\} = Q_1 + Q_2 + Q_3 \quad (38)$$

We shall show shortly that  $[Q_M]$  behaves like an  $[M]$  Stokes matrix, although it obviously is not physically equivalent to a target scattering matrix. This is an example of a "higher order matrix of type  $[M]$ ".

We now conclude our search for conditions imposed upon any Stokes matrix  $[M]$  which claims to represent a single (coherent) target. From (29) we find that the complete matrix  $[Q_M]$  defined by (37) must equal zero! Hence all terms (31), (33), (34) and (35) must vanish. However, not all these conditions are independent, as is easily verified; only four conditions are independent.

In particular, equations (33), (34) and (35), set equal to zero, reveal basic information related to target structure; we may call these the target structure equations. For example, let us assume that  $A_0 = 0$  in (33) and (34), then it must follow (because  $Q_1 = Q_2 = 0$ ) that  $C = D = G = H = 0$ ! Also, from these same equations we find that  $B_0 - B$  and  $B_0 + B$  are non-negative. Hence we find if  $B_0 - B = 0$ , or  $B_0 = B$ , then from (34) and (35) it follows (since  $Q_2 = Q_3 = 0$ ) that  $E = F = G = H$  must be the case. Finally, if  $B_0 + B = 0$ , then by (33) and (35):  $E = F = C = D = 0$ . For these reasons the diagonal elements  $A_0$ ,  $B_0 + B$ , and  $B_0 - B$  are

called the generators of the off-diagonal Stokes parameters.  $A_0$  is the generator of target symmetry.  $B_1 = (B_0 - B)/2$  is, in general, the generator of target nonsymmetry (if  $B_1 = 0$  or  $B_0 = B$  we have a symmetric target), while  $B_2 = (B_0 + B)/2$  is, in general, the generator of target irregularity (if  $B_2 = 0$ , the target is regular). From these two definitions we have

$$B_0 = B_1 + B_2 \quad (39)$$

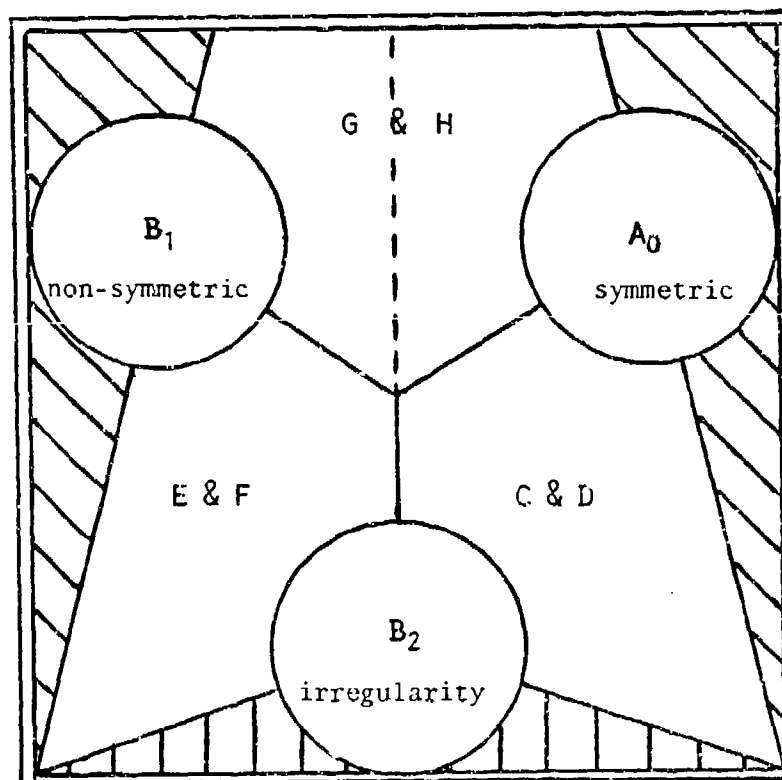
which again emphasizes that  $B_0$  is the sum total of non-symmetric and irregular target components. We are now ready to discuss the complete single target structure.

#### 10. Single Target Structure Diagram

We are now ready to assemble all pieces of information, obtained thus far, on single target parameters, into a complete structure diagram (see next page). The diagram shows a threefold symmetry between target parameters. The three structure generators are  $A_0$ , the generator of target symmetry,  $B_1$ , the generator of target non-symmetry, and  $B_2$ , the generator of target irregularity.

Each generator is responsible for (generates) two pairs of adjacent off-diagonal parameters. Thus  $A_0$  generates the pair C&D and G&H. We already mentioned C&D. The pair G&H are coupling terms.  $H$  is a measure of coupling due to target orientation  $\psi$ . We found that if  $\psi=0$ , then  $H=0$ , whereas  $G$  couples the symmetric and nonsymmetric parts of the target: if  $G=0$  (with  $\psi=0$ ), then either the target is purely symmetric or nonsymmetric.

If  $A_0=0$ , then  $C=D=G=H=0$  and we have the class of non-symmetric N-targets. N-targets play an important role in the theory of distributed targets. There they represent "residue" or target noise at the higher frequencies. N-targets produce the most asymmetric type of scattering (large helicity,  $\tau_m = \pm 45^\circ$ ), such as produced by troughs, edged interacting surfaces, helices, etc. The single N-target is given by four parameters:  $B_0 \geq 0$ ,  $B$ ,  $E$ , and  $F$ , for which  $B_0^2 = B^2 + E^2 + F^2$  (or  $Q_3^N = 0$ ).



TARGET STRUCTURE DIAGRAM  
FOR SINGLE RADAR TARGET

For a target 'on-axis', with  $\psi=0$ , the three generators have the following interesting relationships:

$$A_0 = Q f \cos^2 2\tau_m \quad (40)$$

$$B_1 = Q f \sin^2 2\tau_m \quad (41)$$

$$B_2 = Q(\cos^2 2\gamma + \sin^2 2\gamma \sin^2 2\nu) \quad (42)$$

with parameters defined previously in (13). The first two equations are indicative for target symmetry, or non-symmetry. Equation (42) shows that  $B_2=0$ , when the target is regular only if  $\gamma=45^\circ$  and  $\nu=0^\circ$ , which defines convex or specular type of scattering. It is not clear, at this point, whether target regularity always is equivalent to the specular case.

The target structure diagram is not to be confused with the target decomposition into single target and N-target components, which is discussed next. The diagram reveals the internal structure relationships between Stokes matrix components for a general single (coherent) object.

The insight thus gained into a single target's structural components should greatly benefit our search for meaningful and general target discriminants, which are derived from the electromagnetic interaction with the various classes of target shapes, one wishes to identify.

## 11. Theory of Distributed Radar Targets

When targets are varying with time, and most targets do, we consider the time varying signal:

$$V(t) = [S(t)] \underline{h}_T \cdot \underline{h}_R$$

Now the change of signal with time may be due to a single target's continuous changes in position, or it may be the result of an underlying random process,

involving a large number of objects (chaff). At high frequencies the two processes may, in fact, become intermixed.

For a random process, it is customary to take the expected value of received power:  $\langle P(t) \rangle = \langle [M(t)] \rangle g \cdot h$  as a measure of average RCS. The average Stokes return vector  $\langle \Delta(t) \rangle = \langle [M(t)] \rangle g = Rg$ , in this case no longer will represent a completely polarized (c.p.) wave, instead it will become partially polarized, for which

$$\Delta_0^2 \geq \Delta_1^2 + \Delta_2^2 + \Delta_3^2 \quad (43)$$

holds (we omitted averaging brackets).

We can substitute  $\Delta = [R]g$  into (43) such that:

$$\begin{aligned} \Delta_0^2 - \Delta_1^2 - \Delta_2^2 - \Delta_3^2 &= [Z]\Delta \cdot \Delta \\ &= [R][Z][R] g \cdot g = [Q]g \cdot g = [Q_R]g \cdot g \geq 0 \end{aligned} \quad (44)$$

In the last step we subtracted, as usual, a term  $\alpha[Z]$  from  $[Q]$ , such that the trace rule holds:

$$\text{trace } \{[Q_R]\} = Q_1 + Q_2 + Q_3 \quad (45)$$

Now, let  $[Q_R]g = s = [\Delta_0, \underline{\Delta}]$ , then condition (44) requires that

$$\Delta_0 g_0 + \underline{\Delta} \cdot \underline{g} = \Delta_0 g_0 + |\underline{\Delta}| g_0 \cos \nu \geq 0 \quad (46)$$

for all values of the angle  $\nu$  between vectors. Hence, we find back the familiar

condition  $\Delta_0 \geq |\underline{\Delta}|$  as the basic condition which  $[Q_R]g = s$  has to satisfy.

But, this was exactly the same condition which  $\Delta = [R]g$  had to satisfy, with matrix  $[R]$  replaced by  $[Q_R]$ . Hence,  $[Q_R]$  is called a matrix 'of type  $[R]$ '.

We found before that  $A_0 > 0$ ,  $B_1 > 0$  and  $B_2 > 0$  were necessary conditions for  $[R]$  to be "of type  $[R]$ ". By the same argument, it thus follows that for  $[Q_R]$  we must have:  $Q_1 > 0$ ,  $Q_2 > 0$ , and  $Q_3 > 0$ , because these terms play the same role for  $[Q_R]$  that  $A_0$ ,  $B_1$ , and  $B_2$  play for the matrix  $[R]$ . (Notice the difference with a single target for which  $Q_1 = Q_2 = Q_3 = 0$ !).

We can go one step further to show that if  $\underline{s} = [Q_R] \underline{g}$  and  $s_0 \geq |\underline{s}|$ , then, similarly, as in (44), we must have:

$$\begin{aligned} \Delta_0^2 - \Delta_1^2 - \Delta_2^2 - \Delta_3^2 &= [Z] \Delta \cdot \Delta = [Q_R][Z][Q_R] g \cdot g = \\ &= [Q^{(2)}] g \cdot g = [Q_R^{(2)}] g \cdot g \geq 0 \end{aligned} \quad (47)$$

As before, the last step involves subtraction of a term with a  $Z$  from  $[Q^{(2)}]$  such that:

$$\text{trace } [Q_R^{(2)}] = Q_1^{(2)} + Q_2^{(2)} + Q_3^{(2)} \quad (48)$$

must hold. We then show that  $[Q_R^{(2)}]$  is a higher order matrix of type  $[R]$ . This process can be continued indefinitely, thus producing successfully higher order matrices of type  $R$ .

Fortunately, it turns out that we do not have to go beyond the  $[R]$  and  $[Q_R]$  matrices, because the following, most remarkable relationships were found to exist between higher order matrices of type  $[R]$ :

$$[Q_R^{(2)}] = \chi [R] \quad (49)$$

$$[Q_R^{(3)}] = \chi^2 [Q_R] \quad (50)$$

$$[Q_R^{(4)}] = \chi^5 [R] \quad (51)$$

$$[Q_R^{(5)}] = \chi^{10} [Q_R] \quad (52)$$

where  $\chi$  is a factor of third order in Stokes matrix parameters:

$$\chi = 2A_0 Q_3 + (B_0 - B)Q_1 + (B_0 + B)Q_2 + 2EQ_{23} - 2FQ_{01} \quad (53)$$

Hence, in general:

$$[Q_R^{(2n)}] = \chi^{\frac{1}{3}(2^{2n} - 1)} [R] \quad (54)$$

$$[Q_R^{(2n+1)}] = \chi^{\frac{2}{3}(2^{2n} - 1)} [Q_R] \quad (55)$$

For all these matrices, we have for the three diagonal terms:

$$\boxed{Q_i^{(n)} \geq 0} \quad i = 1, 2, 3 \quad (56)$$

and thus, from (49)

$$\boxed{\chi \geq 0} \quad (57)$$

Here we obtained a deeplying property, which is common to all distributed target matrices (for single targets, we found  $Q_R=0$ ), which reveals its basic structure, as we shall see shortly. The mysterious factor  $\chi$  is an invariant with respect to changes of indices 1, 2 and 3. Thus, all following forms are equivalent expressions, as can be verified by direct calculation, using (31 and 33-35)

$$\begin{aligned} \chi &= -2A_0 Q_3 + (B_0 - B)Q_1 + (B_0 + B)Q_2 + 2EQ_{12} - 2FQ_{03} \\ &= 2A_0 Q_3 - (B_0 - B)Q_1 + (B_0 + B)Q_2 + 2DQ_{23} - 2CQ_{01} \\ &= 2A_0 Q_3 + (B_0 - B)Q_1 - (B_0 + B)Q_2 + 2GQ_{13} - 2HQ_{02} \end{aligned} \quad (58)$$

Notice, that because  $E=R_{12}$ ,  $F=R_{03}$ , etc. we could have obtained a symmetrical notation, if we introduce:

$$A_0 = R_3, \quad B_1 = \frac{B_0 - B}{2} = R_1 \quad \text{and} \quad B_2 = \frac{B_0 + B}{2} = R_2,$$

then the above equation would read:

$$\chi = 2 (R_1 Q_1 + R_2 Q_2 - R_3 Q_3 + R_{12} Q_{12} - R_{03} Q_{03}) \quad (59)$$

with interposition of indices. This notation has some merit, but it also seems better to distinguish individually between the three generators, because of the roles they play in the target structure. For example,  $R_3 = A_0$  is orientation-invariant, but not  $R_1$  or  $R_2$ .

The defining equations for  $Q_{ij}$  in (31) could have been written similarly as:

$$\begin{aligned} Q_1 &= R_2 R_3 - (R_{23}^2 + R_{01}^2) \geq 0 \\ Q_2 &= R_1 R_3 - (R_{13}^2 + R_{02}^2) \geq 0 \\ Q_3 &= R_1 R_2 - (R_{12}^2 + R_{03}^2) \geq 0 \end{aligned} \quad (60)$$

Also, for  $n=2$  we have:

$$\begin{aligned} Q_1^{(2)} &= Q_2 Q_3 - (Q_{23}^2 + Q_{01}^2) \geq 0 \\ Q_2^{(2)} &= Q_1 Q_3 - (Q_{13}^2 + Q_{02}^2) \geq 0 \\ Q_3^{(2)} &= Q_1 Q_2 - (Q_{12}^2 + Q_{03}^2) \geq 0 \end{aligned} \quad (61)$$

The last expression will be used with the target decomposition theorems, to be discussed next.

#### TARGET DECOMPOSITION THEOREMS

##### 12. Irreducibility of Elliptically Polarized Waves

As is well known, the Stokes vector  $\delta = (\delta_0, \underline{\delta}) = (\delta_0, \delta_1, \delta_2, \delta_3)$  for an

elliptically polarized wave (e.p.) satisfies the condition  $\Delta_0^2 = \Delta_1^2 + \Delta_2^2 + \Delta_3^2$ , or  $\Delta_0 = |\underline{\Delta}|$ , whereas for a partially polarized (p.p.) wave  $p = (p_0, \underline{p})$  we must have  $p_0 \geq |\underline{p}|$ . It is easy to see that the partially polarized case can always be written as the incoherent sum of an e.p. wave  $\Delta = (\Delta_0, |\underline{\Delta}|)$ , with  $\underline{\Delta} = \underline{p}$ , and a completely unpolarized (u.p.) wave:  $n = (n_0, 0, 0, 0)$ :  $p = \Delta + n$ , where  $n_0 = p_0 - \Delta_0 = p_0 - |\underline{\Delta}| = p_0 - |\underline{p}|$ . Hence, if  $p_0 = |\underline{p}|$ , we find that  $n_0 = 0$ ; hence, it follows that an elliptically polarized wave is irreducible; i.e. it cannot be separated in any way further into the independent sum of physically realizable parts. In that sense it has an atomic character.

The decomposition of the p.p. Stokes vector satisfies the intuitive demand to consider the p.p. wave as the independent sum of an irreducible object (the e.p. part) and a noise component (the u.p. part). This process brings about a new method, by which one can separate a complex signal into "pure signal" and "noisy signal", with attendant signal processing. Notice that the noisy part has no "structure", if by "structure" is meant the vector component of the Stokes vector.

In this way, a generalized Stokes vector  $p = (p_0, \underline{p}) = (p_0, p_1, p_2, \dots, p_n)$  with  $p_0 = |\underline{p}|$ , can be viewed as representing an object, which is given by  $n$  measurable parameters, which determine object-structure, and  $p_0$ , which manifests object irreducibility, object integrity, individuality or existence.

We can thus expect the generalized Stokes vector to have important applications in such widely diverse fields as: pattern recognition, general cognition,

theory of corporations, cell biology, linguistics, perception, and classical and quantum mechanics (where it gives the Hamiltonian).

### 13. Basic Irreducibility of Single Radar Targets

Some of the ideas presented above for Stokes vectors can be carried over to radar targets. The basic idea is that of an individual or single target. We found before that a single target (or coherent target), when illuminated by an e.p. wave, scatters as an e.p. wave. We found above that the scattered e.p. wave is irreducible, i.e. it cannot be split into an incoherent or independent sum of physically realizable component parts.

From this argument, it follows at once that the single target itself must have the irreducible property. For a direct proof of this important result, see [ 3 ], page 157. In other words, the single target must possess the property of atomicity or individuality, which is basic to the single object scattering structure. This theorem points to a fundamental limitation of traditional attempts at "sectionalizing" a single object of complex shape, such as an airplane, into independent simpler shapes, like wings, tail, fuselage, etc. Although at higher frequencies, when size/wavelength is large and coupling is small, such methods for computing RCS have had some success; the theorem shows the futility of such attempts at the lower frequencies.

The theorem obviously also has profound impact on other areas of science mentioned above. It shows a basic irreducibility of objects of perception, atomicity of elementary particles, and in quantum theory, the whole universe may be viewed as one big irreducible object, i.e. everything is fundamentally connected (Bell's inseparability theorem). In quantum theory, an irreducible subsystem is called a "pure case" as contrasted to a "mixture" of pure cases.

We thus find how such diverse sciences all have a common basic structure which derives from the Stokes vector concept. This seems to be a new fundamental insight, which has not previously been explored in the open literature.

#### 14. Decomposition of Distributed Targets

We found previously that given aspect, frequency and waveform, a general distributed target with Stokes matrix  $[R]$  has nine independent parameters, whereas a single target, with Stokes matrix  $[M_0]$  has five independent parameters. It thus seems natural to consider the possibility of decomposing the nine-parameter target structure  $[R]$  into an average single effect object  $[M_0]$  (given by five parameters) and a residue  $[N]$ -target, which contains the four remaining degrees of freedom:

$$[R(9)] = \langle [M(t)] \rangle = [M_0(5)] \cdot [N(4)] \quad (62)$$

We have chosen the  $N$ -target for residue because it possesses the highest degree of non-symmetry and irregularity and it is determined by four parameters only:  $B_0^N > 0$ ,

$B^N$ ,  $E^N$ , and  $F^N$ , for which:  $B_o^{N^2} \geq B^{N^2} + E^{N^2} + F^{N^2}$  must hold.

Even more important: the class of N-targets does not depend on changes in target or radar observation orientation. In other words, a change in orientation,  $\psi$ , produces another N-target of the same class. Hence, the N-target is an excellent candidate to represent target residue, or target noise, which is due to the splitting off of the single average target from the distributed target.

We can extend the concept not only to power averages, but indeed to the instantaneous return signals as well. At each instant the return from a time varying target can be split into components which contribute exclusively to an "effective target" and components which contribute exclusively to the N-target residue; i.e. one can show that:  $[T(t)] = a(t)[T_o] + [T_N(t)]$ , (63) where  $[T_o]$  is a constant matrix for the effective target. Thus the incoming target return is decomposed into a desirable part which is "signal" and a residue term which is "target noise". The decomposition makes it possible to focus on the "effective target" as effectively representing the ensemble.

The physical realizability of the decomposition theorem is proved in Appendix A. We now summarize this section by providing a list of target decompositions and properties.

- A. General decomposition theorem: A general distributed target  $[R]$  is uniquely decomposed into a single average target  $[M_0]$  plus distributed N-target residue

$$[R] = [M_0] + [N(\text{distr.})]$$

- B. A general distributed target  $[R]$  is decomposable (not unique) into three independent single targets

$$[R] = [M_0] + [M_2] + [M_3]$$

- C. The sum of two independent single targets is uniquely decomposable into a single target plus a single N-target:

$$[R] = [M_1] + [M_2] = [M_0] + [N(\text{single})]$$

The criterion for this case is  $\chi = 0$ .

- D. A single (coherent) target is irreducible and has five independent parameters, the condition of which is given by:  $[Q_M] = 0$ .

- E. Any distributed N-target has a (non-unique) decomposition as a sum of two single N-targets:

$$[N(\text{distr.})] = [N_1(\text{single})] + [N_2(\text{single})]$$

#### 15. The Orientation-Independent Target

This target model is the simplest of all possible non-single models; it is given by just two parameters:  $A_0 \geq 0$  and  $B_0 \geq 0$ . This model has no preferred orientation  $\psi$  and hence there is no preferred orientation bias. For terrain this represents plains and fields viewed at close to normal direction but not a mountain ridge or river bed. This model is useful even to describe targets

which in fact have a preferred orientation, because in this case, since there is usually uncertainty about its direction, one can treat the orientation of the target as a random variable which has equal distribution in all directions. With that provision also these targets become orientation independent [4]. Examples of orientation-independent targets (not to be confused with orientation independent target parameters) are homogeneous terrain and sea state surface viewed from close to normal incidence. Also homogeneous clouds of rain, dust particles, and chaff. The Stokes matrix is given by:

$$\begin{aligned}
 [R] &= \begin{bmatrix} A_0 + B_0 & A_0 \\ A_0 & -A_0 + B_0 \end{bmatrix} = [M_0] + [N] = \\
 &= A_0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + B_1 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + B_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (64)
 \end{aligned}$$

where  $B_0 = B_1 + B_2$

In this case the "effective target" is associated with  $A_0$  (specular scattering or a sphere) while the N-target residue consists of the sum of "trough noise" type of returns (in this case  $B_1 = B_2$ ). At each instant in time, the corresponding scattering matrix decomposition will be:

$$[S(t)] = a(t) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b(t) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + c(t) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} a+b & c \\ c & a-b \end{bmatrix} \quad (65)$$

where the elements  $a(t)$ ,  $b(t)$  and  $c(t)$  are uncorrelated.

The scattering due to the N-target, for this case, is interesting.

For any linear transmit polarization we find:

$$[N] \begin{bmatrix} 1 \\ p \\ q \\ 0 \end{bmatrix} = \begin{bmatrix} B_o & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_o \end{bmatrix} \begin{bmatrix} 1 \\ p \\ q \\ 0 \end{bmatrix} = \begin{bmatrix} B_o \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (66)$$

and thus the scattering is unpolarized. However, this result is not true for circular transmit polarizations.

#### 16. Summary

This report highlights some results of a phenomenological approach to radar targets, with applications. The approach grew out of the common sense realization that only those target data are acceptable for discrimination and identification purposes which can be shown to relate in a physically meaningful way to basic target structure. Only then can data, often gathered at great expense, obtained for one type of system, be expected to be useful productively for a new system and hence improve efficiency and cost factors.

Although these comments are almost self-evident and common sensical in nature, examples are given to show how this systematic approach has an important effect on the mathematical and practical development towards target identification (inverse) problems. The effect of antenna and target orientation angle (Fig. 1) on corrupting target information is stressed, in contrast to common practice to allow single H or V polarization data to be accepted as meaningful. So-called orientation-independent target parameters are derived

from the target Stokes matrix which allows for physical interpretation and correlation with target structure.

The report summarizes the general target decomposition theorems, proved by the author in 1970. It shows that a single-coherent object is electromagnetically irreducible (it cannot be broken down mathematically as the incoherent sum of the smaller parts without violating physical principles). A general distributed target (such as chaff or surface return) can be broken down into the (incoherent) sums of a "single average object" and "N-target" residue. The latter may be considered as a form of "target noise".

All this opens up new vistas for optimal signal processing schemes which extend the present predominantly scalar case to include vector scattering problems. It is hoped that by these efforts improved reliability with reduced costs for target discrimination and identification purposes can be achieved.

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APPENDIX A

Proof of Target Decomposition Theorem:  $[R] = [M] + [N]$

The N-target is given by four independent parameters  $B_O^N$ ,  $B^N$ ,  $E^N$ , and  $F^N$ .

Two conditions:  $B_O^N \geq 0$  and  $Q_3^N \geq 0$  have to be satisfied for the N-target to be physically realizable.

The general distributed target  $[R]$  is given by nine parameters  $A_O \geq 0$ ,  $B_O$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $F$ ,  $G$ , and  $H$ . Hence  $B_O = B_O^T + B_O^N$ ,  $B = B^T + B^N$ ,  $E = E^T + E^N$ ,  $F = F^T + F^N$ .

For  $[M]$  to be a single target, four conditions are imposed on the nine single target parameters  $A_O$ ,  $B_O^T$ ,  $B^T$ ,  $C$ ,  $D$ ,  $E^T$ ,  $F^T$ ,  $G$  and  $H$ .

These are:

$$Q_1^T = 2A_O(B_O^T + B^T) - (C^2 + D^2) = 0 \quad (A1)$$

$$Q_2^T = 2A_O(B_O^T - B^T) - (G^2 + H^2) = 0 \quad (A2)$$

$$Q_{12}^T = -2A_O E^T + (CH - DG) = 0 \quad (A3)$$

$$Q_{03}^T = 2A_O F^T - (CG + DH) = 0 \quad (A4)$$

First we show  $B_O^N \geq 0$ :

$$\begin{aligned} 4A_O B_O^N &= 4A_O B_O - 4A_O B^T = 4A_O B_O - (C^2 + D^2 + G^2 + H^2) \\ &= [2A_O(B_O + B) - (C^2 + D^2)] + [2A_O(B_O - B) - (G^2 + H^2)] \\ &= Q_1 + Q_2 \geq 0 \end{aligned} \quad (A5)$$

In the second step in (A5), the index T variables were eliminated by combining (A1) and (A2). The last step follows from the definition for  $Q_1$  and  $Q_2$  (33 and 34) and (56) for  $n=1$ . Because  $A_O \geq 0$ , we have shown that  $B_O^N \geq 0$ .

APPENDIX A  
Continued

The second part of the proof follows similar lines:

We wish to prove  $Q_3^N \geq 0$ , where

$$\begin{aligned}
 Q_3^N &= B_0^N - B^N - E^N - F^N = \\
 &= (B_0 - B_0^T)^2 - (B - B^T)^2 - (E - E^T)^2 - (F - F^T)^2 = \\
 &= (B_0^2 - B^2 - E^2 - F^2) + (B_0^{T^2} - B^{T^2} - E^{T^2} - F^{T^2}) + \\
 &\quad - 2(B_0 B_0^T - BB^T - EE^T - FF^T) \tag{A6}
 \end{aligned}$$

The second term in (A6) is zero:  $Q_3^T = 0$  for a single object.

Hence:

$$Q_3^N = Q_3 - 2(B_0 B_0^T - BB^T - EE^T - FF^T) \tag{A7}$$

Next, we multiply with  $4A_0^2$  and eliminate index T variables by using relationships (A1, A2, A3, and A4)

$$\begin{aligned}
 4A_0^2 Q_3^N &= 4A_0^2 (B_0^2 - B^2 - E^2 - F^2) - 2A_0 B_0 (C^2 + D^2 + G^2 + H^2) + \\
 &\quad + 2A_0 B (C^2 + D^2 - G^2 - H^2) + 4A_0 E (CH - DC) + 4A_0 F (CG + DA) \\
 &= [2A_0 (B_0 + B) - (C^2 + D^2)] [2A_0 (B_0 - B) - (G^2 + H^2)] + \\
 &\quad - [2A_0 F - (CG + DH)]^2 - [-2A_0 E + (CH - DG)]^2 \tag{A8}
 \end{aligned}$$

$$\begin{aligned}
 \text{or: } 4A_0^2 Q_3^N &= Q_1 Q_2 - [Q_{03}^2 + Q_{12}^2] = \\
 &= Q_3^{(2)} = 2A_0 x \geq 0 \tag{A9}
 \end{aligned}$$

The last steps follow from (61-3) and (57).

We thus discover that the deeplying property  $x \geq 0$ , lies at the heart of the target decomposition theorem.

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